

Effective Field Theory Calculation of nd Radiative Capture at Thermal Energies

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Abstract

The cross section for the thermal neutron capture by the deuteron is calculated with pionless Effective Field Theory(EFT). No new Three-Nucleon forces are needed up to next-to-next-to-leading order in order to achieve cut-off independent results, besides those fixed by the triton binding energy and Nd scattering length in the triton channel. The cross-section is accurately determined to be $\sigma_{tot} = [0.503 \pm 0.003]mb$. At zero energies, the magnetic $M1$ -transition gives the dominant contribution and is calculated up to next-to-next-to-leading order (N^2LO). Close agreement between the available experimental data and the calculated cross section is reached. We demonstrate convergence and cutoff independence order by order in the low-energy expansion.

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I. INTRODUCTION

The study of the three-body nuclear system involving neutron radiative capture by deuteron has been investigated in theoretical and experimental works over the past years. The experimental result of this process has most accurately been measured by Journey, et.al. [1]. The value of $0.508 \pm 0.015(\text{mb})$ for the cross section was resulted for 2200 m/sec neutrons.

Rapid progress has been made in the theoretical study of the $Nd \rightarrow {}^3H\gamma$ reaction such as the p - d and n - d radiative capture. At such energies a magnetic dipole(M_1)transition is almost entirely participated. These reactions were studied in plane wave (Born) approximation by Friar et al. [2]. In these investigations the authors employed their configuration-space Faddeev calculations of the helium wave function, with inclusion of three-body forces and pion exchange currents. More recently a rather detailed investigation of such processes has been performed by Viviani et al. [3, 19]. In their calculations the quite accurate three-nucleon bound- and continuum states were obtained in the variational pair-correlated hyperspherical method from a realistic Hamiltonian model with two- nucleon and three-nucleon interactions.

They obtained in Ref. [3] the cross section from Argonne v_{14} two-nucleon and Urbana VIII three-nucleon interactions(AV14/UVIII), also from Argonne v_{18} two-nucleon and Urbana IX three-nucleon interactions(AV18/UIX) and including Δ admixtures. Cross section values were found 0.600 (mb) and 0.578 (mb) which overestimate the experimental value by 18% and 14% value, respectively, see table 2. It should be noted, however, that the explicit, non-perturbative inclusion of Δ -isobar degrees of freedom in the nuclear wave function are found to be in significantly better agreement with experiment than those obtained from perturbative (Δ_{PT}) estimates. This shows that these results for this very-low energy observable are sensitive to details of the short-range part of the interaction. recent calculation using manifestly gauge-invariant currents reduced the spread [19], but the result including three-body currents, 0.558 mb, still over-predicts the cross-section by 10%. Model-dependent currents associated with the $\Delta(1232)$ were identified as source of the discrepancy. Thus, the question remains how such details of short-range Physics can so severely influence a very-long-range reaction with maximal energies of less than 10 MeV.

During the last few years, nuclear Effective Field Theory(EFT) has been applied to two-, three-, and four-nucleon systems, see e.g. [4, 5, 6, 7, 8, 9, 10]. The pionless Effective Field Theory would be an ideal tool to calculate low-energy cross sections in a model-independent way and to possibly reduce the theoretical errors by a systematic, model-independent calculation with an a-priori estimate of the theoretical uncertainties. An example of a precise calculation is the reaction $np \rightarrow \gamma d$, which is relevant to big-bang nucleosynthesis(BBN). The cross section for this process was computed to 1% error for center of mass energies $E \lesssim 1\text{MeV}$ [11, 12, 13].

We have suggested a method for computation of neutron-deuteron radiative capture for extremely low energy($20 \leq E \leq 200$ Kev)with pionless EFT [15], where with this formalism, we can estimate errors in a perturbative expansion up to N²LO within a few percent of the ENDF values [16].

The purpose of the present paper is to study the cross section for radiative capture of

neutrons by deuterons $nd \rightarrow \gamma^3 H$ at zero energies with pionless EFT. At these energies, the magnetic M_1 -transition gives the dominant contribution. The M_1 amplitude is calculated up to next-to-next-to-leading order (N²LO) with insertion of three body force. Results show less than 1% deviation from the available experimental data at zero energy (0.0253 eV).

This article is organized as follows. In the next section, a brief description of the formalism and its input for total cross section of the neutron-deuteron radiative capture will be presented. We discuss the theoretical errors, tabulation of the calculated cross section in comparison with the other theoretical approaches and the most recent data [1] in section III. Finally, Summary and conclusions follow in Section IV.

II. NEUTRON-DEUTERON SCATTERING IN TRITON CHANNEL AND RADIATIVE CAPTURE

The $^2S_{1/2}$ channel to which ^3He and ^3H belong is qualitatively different from the other three-nucleon channels because all three nucleons can occupy the same points in space. Consequently, $^2S_{1/2}$ describes the preferred mode for $nd \rightarrow ^3H\gamma$ and $pd \rightarrow ^3He\gamma$. The three-nucleon Lagrangean is well-known and will not be repeated here, see e.g. [14,18] for details.

The derivation of the integral equation describing neutron-deuteron scattering has also been discussed before, see e.g. [7, 18]. We present here only the results. The integral equation is solved numerically by imposing a cut-off Λ . In that case, a unique solution exists in the $^2S_{1/2}$ -channel for each Λ and vanishing three-body force, but no unique limit as $\Lambda \rightarrow \infty$. As long-distance phenomena must however be insensitive to details of the short-distance physics (and in particular of the regulator chosen), Bedaque et al. [6, 7, 14, 18] showed that the system must be stabilized by a three-body force

$$\mathcal{H}(E; \Lambda) = \frac{2}{\Lambda^2} \sum_{n=0}^{\infty} H_{2n}(\Lambda) \left(\frac{ME + \gamma_t^2}{\Lambda^2} \right)^n = \frac{2H_0(\Lambda)}{\Lambda^2} + \frac{2H_2(\Lambda)}{\Lambda^4} (ME + \gamma_t^2) + \dots \quad (1)$$

which absorbs all dependence on the cut-off as $\Lambda \rightarrow \infty$. It is analytical in E and can be obtained from a three-body Lagrangean, employing a three-nucleon auxiliary field analogous to the treatment of the two-nucleon channels [14]. Contrary to the terms without derivatives, there are different, inequivalent three-body force terms with *two* derivatives, but only one of them, H_2 , is enhanced over its naive dimensional estimate, mandating its inclusion at N²LO [14,20]. Neutron-deuteron scattering amplitude including the new term generated by the two-derivative three-body force is shown schematically in Fig.1. Two amplitudes get mixed: t_s describes the $d_t + N \rightarrow d_s + N$ process, and t_t describes the $d_t + N \rightarrow d_t + N$ process, where d_s (d_t) is an auxiliary field of two nucleons in a relative singlet-S (triplet-S) wave.

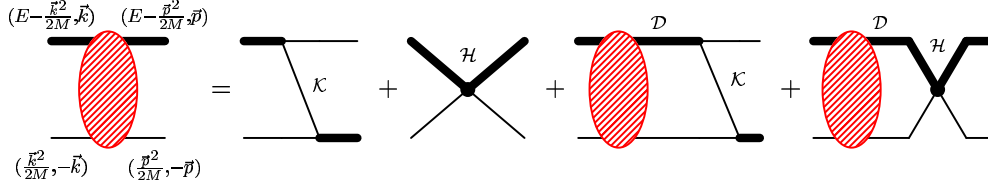


FIG. 1: The Faddeev equation for Nd -scattering. Thick solid line is propagator of the two intermediate auxiliary fields D_s and D_t , denoted by \mathcal{D} ; \mathcal{K} : propagator of the exchanged nucleon; \mathcal{H} : three-body force.

$$\begin{aligned}
t_s(p, k) &= \frac{1}{4} [3\mathcal{K}(p, k) + 2\mathcal{H}(E, \Lambda)] + \frac{1}{2\pi} \int_0^\Lambda dq \, q^2 [\mathcal{D}_s(q) [\mathcal{K}(p, q) + 2\mathcal{H}(E, \Lambda)] t_s(q) \\
&\quad + \mathcal{D}_t(q) [3\mathcal{K}(p, q) + 2\mathcal{H}(E, \Lambda)] t_t(q)] \\
t_t(p, k) &= \frac{1}{4} [\mathcal{K}(p, k) + 2\mathcal{H}(E, \Lambda)] + \frac{1}{2\pi} \int_0^\Lambda dq \, q^2 [\mathcal{D}_t(q) [\mathcal{K}(p, q) + 2\mathcal{H}(E, \Lambda)] t_t(q) \\
&\quad + \mathcal{D}_s(q) [3\mathcal{K}(p, q) + 2\mathcal{H}(E, \Lambda)] t_s(q)] \quad (2)
\end{aligned}$$

where $\mathcal{D}_{s,t}(q) = \mathcal{D}_{s,t}(E - \frac{q^2}{2M}, q)$ are the propagators of the auxiliary fields $d_{s,t}$, and \mathcal{K} the propagator of the exchanged nucleon, projected into the S-wave. For the spin-triplet S-wave channel, one determines the two-nucleon interaction up to N²LO by the deuteron binding momentum $\gamma_t = 45.7025$ MeV and effective range $\rho_t = 1.764$ fm. Because there is no real bound state in the spin singlet channel of the two-nucleon system, its free parameters are better determined by the scattering length $a_s = 1/\gamma_s = -23.714$ fm and the effective range $r_s = 2.73$ fm at zero momentum.

The neutron-deuteron $J = 1/2$ phase shifts δ is determined by the on-shell amplitude $t_t(k, k)$, multiplied with the wave function renormalisation

$$T(k) = Z t_t(k, k) = \frac{3\pi}{M} \frac{1}{k \cot \delta - ik} \quad (3)$$

At thermal energies, the reaction proceeds through S-wave capture predominantly via a magnetic dipole transition, M_i^{LSJ} , where $L=0$, $S=1/2, 3/2$ and $i=1$. To obtain the spin structure, which corresponds to a definite value of J for the entrance channel, it is necessary to build special linear combinations of products $\vec{D}N$ and $\vec{\sigma} \times \vec{D}N$, with $J^P = \frac{1}{2}^+$ or $J^P = \frac{3}{2}^+$, and \vec{D} the deuteron spin-one field, see [15] for details.

$$\vec{\phi}_{1/2} = (i\vec{D} + \vec{\sigma} \times \vec{D})N \text{ and } (2i\vec{D} - \vec{\sigma} \times \vec{D})N.$$

For both possible magnetic dipole transitions with $J^P = \frac{1}{2}^+$ (amplitude g_1) and $J^P = \frac{3}{2}^+$

(amplitude g_3) we can write:

$$g_1 : t^\dagger (i\vec{D} \cdot \vec{e}^* \times \vec{k} + \vec{\sigma} \times \vec{D} \cdot \vec{e}^* \times \vec{k}) N,$$

$$g_3 : t^\dagger (i\vec{D} \cdot \vec{e}^* \times \vec{k} + \vec{\sigma} \times \vec{D} \cdot \vec{e}^* \times \vec{k}) N. \quad (4)$$

The contribution of the electric transition E_i^{LSJ} for energies of less than 60 KeV to the total cross section is very small. Therefore, the electric quadrupole transition $E_2^{0(3/2)(3/2)}$ from the initial quartet state will not be considered at thermal energies. The M_1 amplitude receives contributions from the magnetic moments of the nucleon and dibaryon operators coupling to the magnetic field, which are described by the Lagrange density

$$\mathcal{L}_B = \frac{e}{2M_N} N^\dagger (k_0 + k_1 \tau^3) \sigma \cdot B + e \frac{L_1}{M_N \sqrt{r(^1S_0) r(^3S_1)}} d_t^{j\dagger} d_{s3} B_j + H.C. \quad (5)$$

where $k_0 = 1/2(k_p + k_n) = 0.4399$ and $k_1 = 1/2(k_p - k_n) = 2.35294$ are the isoscalar and isovector nucleon magnetic moment in nuclear magnetons, respectively. The NLO-coefficient L_1 is fixed at its leading non-vanishing order to the thermal cross section [11].

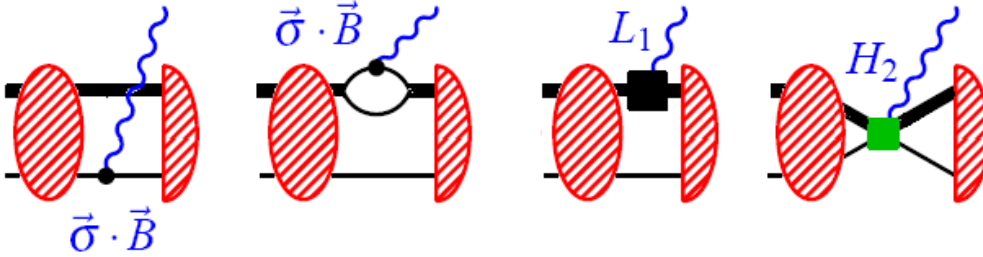


FIG. 2: Some diagrams for adding photon-interaction to the Faddeev equation up to N^2LO . Wavy line shows photon and small circles show magnetic photon interaction. For L_1 vertices, see eq.(7); H_2 :three- body force, see eq.(3). Remaining notation as in Fig. 1.

The radiative capture cross section $nd \rightarrow {}^3H\gamma$ at very low energy is given by

$$\sigma = \frac{2}{9} \frac{\alpha}{v_{rel}} \frac{p^3}{4M_N^2} \sum_{iLSJ} [|\tilde{\chi}_i^{LSJ}|^2], \quad (6)$$

where

$$\tilde{\chi}_i^{LSJ} = \frac{\sqrt{6\pi}}{p\mu_N} \sqrt{4\pi} \chi_i^{LSJ}, \quad (7)$$

with χ stands for either E or M and μ_N is in nuclear magneton and p is momentum of the incident neutron in the center of mass.

We now turn to the Faddeev integral equation to be used in the M_1 calculation. We solve the Faddeev equation for nd -scattering and also for the triton bound state to some order (e.g. LO), then we take these Faddeev amplitudes and sandwich the photon-interactions with

nucleons between them when the photon kernel is expanded to the same order. This process will be done separately for NLO and N²LO. Finally the wave function renormalization in each order will be done.

The diagrams in Fig. 2 represent contributions of electromagnetic interaction with nucleon, deuteron, four-nucleon-magnetic-photon operator described by a coupling between the 3S_1 -dibaryon and 1S_0 -dibaryon and a magnetic photon. As mentioned in the introduction, in another paper [15], we have presented detailed schematic of these diagrams in neutron-deuteron radiative capture for ($20 \leq E \leq 200$ keV) up to N²LO.

The last diagrams in Fig. 2 with insertion of a photon to the N²LO three-nucleon force H_2 vertex is not M_1 and we know that M_1 contribution is the dominant contribution at very low energy and especially for zero energy. Its contribution should therefore be very tiny. Because the leading three-nucleon force H_0 has no derivatives, it is not affected by the minimal substitution $p \rightarrow p - eA$. But the parameter H_2 is the strength of the three-nucleon interaction with two derivatives. Naturally for the energy range near zero momentum, insertion of photon to H_2 vertices for momentum $p \sim 0.025$ eV and M_1 transition, could be neglected. H_2 is necessary in neutron-deuteron scattering to improve cut-off independence but is defined such that it does not contribute at zero momentum. Contributions of a photon coupling to H_2 are however indeed negligible at zero energy.

III. NEUTRON-DEUTERON RADIATIVE CAPTURE RESULTS AT ZERO ENERGY

We numerically solved the Faddeev integral equation up to N²LO. We used $\hbar c = 197.327$ MeV fm, a nucleon mass of $M = 938.918$ MeV, for the NN triplet channel a deuteron binding energy (momentum) of $B = 2.225$ MeV ($\gamma_d = 45.7066$ MeV), a residue of $Z_d = 1.690(3)$, for the NN singlet channel an 1S_0 scattering length of $a_s = -23.714$ fm, $L_1 \sim -4.5$ fm by fixing at its leading non-vanishing order by the thermal cross section.

As in Ref. [20], we can determine which three-body forces are required at any given order, and how they depend on the cutoff.

Low-energy observables must be insensitive to the cut-off, namely to any details of short-distance physics in the region above the break-down scale of the pion-less EFT, set approximately by the pion-mass. It was found in Ref. [20] that no additional three-nucleon forces are necessary to render a renormalisable amplitude at N²LO in this process, besides those needed already in nucleon-deuteron scattering: H_0 and H_2 . At N²LO, where we saw that H_2 is required, we checked this by varying the cut-off between 150 and 500 MeV. This is a reasonable estimate of the errors of our calculation due to higher-order effects. As seen in Fig. 3, in the thermal energy range the cutoff variation is very small and decreases steadily as we increase the order of the calculation and it is of the order of $(k/\Lambda)^n, (\gamma/\Lambda)^n$, where n is the order of the calculation and $\Lambda = 150$ MeV is the smallest cutoff used (see Table I and Fig. 3). Also, errors due to cutoff variation is decreasing when the order of calculation is increased up to N²LO.

We determined the two-nucleon parameters from the deuteron binding energy, triplet effective range (defined by an expansion around the deuteron pole, not at zero momentum), the singlet scattering length, effective range (defined by expanding at zero momentum), and

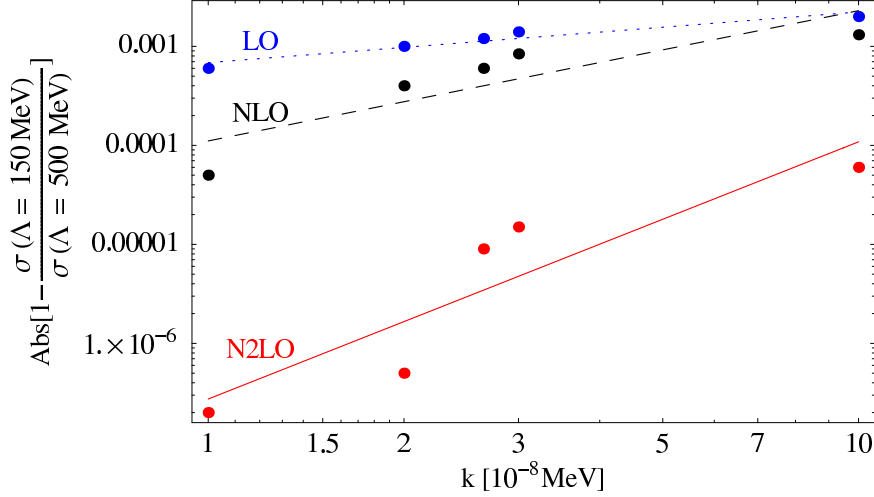


FIG. 3: Curve of the cutoff variation of cross section up to N²LO is shown between $\Lambda = 150$ MeV and $\Lambda = 500$ MeV. The short dashed, long dashed and solid line correspond to LO, NLO and N²LO, respectively.

TABLE I: Results for the cutoff variation of cross section up to N²LO is shown between $\Lambda = 150$ MeV and $\Lambda = 500$ MeV.

E(10 ⁻⁸ MeV)	LO	NLO	N ² LO
1	0.0006	0.00005	0.0000002
2	0.0010	0.00040	0.0000005
2.65	0.0012	0.00060	0.0000090
3	0.0014	0.00084	0.0000150
10	0.0020	0.00131	0.0000600

two body capture process (obtained with comparison between experimental data and theoretical results for $np \rightarrow d\gamma$ process at zero energy [12]). We fix the three-body parameters as follows: because we defined H_2 such that it does not contribute at zero momentum scattering, one can first determine H_0 from the $^2S_{1/2}$ scattering length $a_3 = (0.65 \pm 0.04)$ fm [17]. At LO and NLO, this is the only three-body force. At N²LO, H_2 is required. It is determined by the triton binding energy $B_3 = 8.48$ MeV. Finally, we solve by insertion of the potential at a given order in the integral equation and iteration of kernel.

The cross section for neutron-deuteron radiative capture as function of the center-of-mass energy up to N²LO is shown in Fig. 4. We also show single point that shows the available experimental results for this cross section at 0.025 eV [1].

Table II shows Comparison between results of different models-dependent, model-independent EFT and experiment, for neutron radiative capture by deuteron up to N²LO,

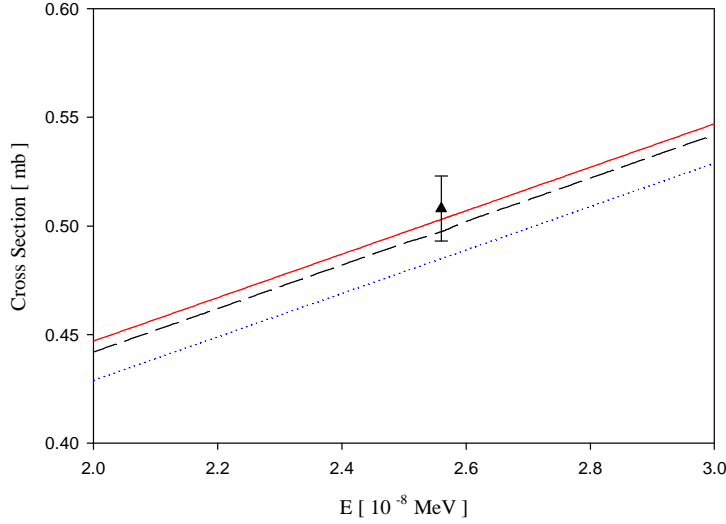


FIG. 4: The cross section for neutron radiative capture by deuteron as function of the center-of-mass kinetic energy E in MeV. The short dashed, long dashed and solid line correspond to the contribution of M_1 capture cross section up to LO, NLO and N^2 LO , respectively. Single point shows experimental results for this cross section at 0.025 eV [1].

TABLE II: Comparison between different theoretical results for Neutron radiative capture by deuteron at zero energy (0.0253 ev). Last row shows our EFT result. The last line quotes deviation between data [1] and theory, if it is larger than the theoretical or experimental uncertainty.

Theory	$\sigma(\text{mb})$	deviation from exp.
AV14/VIII (IA+MI+MD) [3]	0.509	
AV18/IX (IA+MI+MD) [3]	0.489	4%
AV14/VIII(IA+MI+MD+ Δ_{PT}) [3]	0.658	29%
AV18/IX(IA+MI+MD+ Δ_{PT}) [3]	0.631	24%
AV14/VIII(IA+MI+MD+ Δ) [3]	0.600	18%
AV18/IX(IA+MI+MD+ Δ) [3]	0.578	14%
AV18/IX (gauge inv.) [19]	0.523	3%
AV18/IX (gauge inv. + 3N-current) [19]	0.556	
EFT(LO)	0.485	5%
EFT(NLO)	0.496	
EFT(N^2 LO)	0.503 ± 0.003	
Experiment [1]	0.508 ± 0.015	

at zero energy (0.0253 ev). The calculations by Viviani et al. [3, 19] shows sensitivity to

short-range physics namely to details of including the physics of the Delta and pion-exchange currents. The calculation of Ref. [19] with manifestly gauge-invariant current operators is quite sensitive to including meson-exchange three-nucleon currents. One might therefore have been tempted to conclude that a new three-nucleon force is also needed in the pion-less EFT. As shown above, this is not the case: There are no new three-nucleon forces besides those already fixed in nd scattering at the same order. The contribution from the photon coupling to a three-nucleon force is negligible in our calculation. As our result is model-independent and universal, any model with the same input must – within the accuracy of our calculation – lead to the same result. Our inputs are the first two terms of the effective-range expansion in the singlet- and triplet-S wave of NN scattering, the proton and neutron magnetic moments, the triton binding energy and nd scattering length in the doublet-S-wave, and finally the thermal cross section of the reaction $np \rightarrow d\gamma$ (determining L_1). More work is needed to understand why the potential-model calculations [3,19] have the same input but do not seem to reproduce the same result.

Addressing convergence of the EFT calculation, we notice that the contributions which are characterized as higher-order in the power-counting are indeed small: The LO result is 0.485 mb, with NLO adding 0.011 mb, and N²LO another 0.007 mb. Cut-off dependence is negligible. The typical size of the expansion parameter in the pion-less EFT is about $\gamma_t/m_\pi \approx 1/3$. We therefore estimate the uncertainty from leaving out corrections at N³LO and higher as about 1/3 of the N²LO correction or 0.003 mb.

IV. CONCLUSION

The cross section for radiative capture of neutrons by deuterons $nd \rightarrow \gamma^3H$ at zero energies with was calculated pionless Effective Field Theory, the unique, model independent and systematic low-energy version of QCD for processes involving momenta below the pion mass. We applied pionless EFT to find numerical results for the M_1 contributions. Incident thermal neutron energies have been considered for this capture process. At these energy our calculation is dominated by only S -wave state and magnetic transition M_1 contribution. The M_1 amplitude is calculated up to Next-to-Next to leading order N²LO. Three-Nucleon forces are needed up to N²LO order for cut-off independent results. The triton binding energy and nd scattering length in the triton channel have been used to fix them. Hence the cross-section is in total determined as $\sigma_{tot} = [0.485(LO) + 0.011(NLO) + 0.007(N^2LO)] = [0.503 \pm 0.003]mb$. It converges order by order in low energy expansion. It is also cut-off independent at this order. We notice that our calculation has a systematic uncertainty from higher-order terms which is now smaller than the experimental error-bar.

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- [1] E.T. Journey, P.J. Bendt and J.C. Browne, Phys. Rev. C **25**, 2810 (1982).
 - [2] J.L. Friar, B.F. Gibson and G.L. Payne, Phys. Lett. B **251**, 11 (1990).
 - [3] M. Viviani, R. Schiavilla and A. Kievsky, Phys. Rev. C **54**, 534 (1996).
 - [4] D. B. Kaplan, M. J. Savage and M. B. Wise, Nucl. Phys. B **534**, 329 (1998).
 - [5] S. R. Beane and M. J. Savage, Nucl. Phys. A **694**, 511 (2001).
 - [6] P. F. Bedaque, H.-W. Hammer and U. van Kolck, Phys. Rev. Lett. **82**, 463 (1999); Nucl. Phys. A **646**, 444 (1999).
 - [7] P. F. Bedaque, H.-W. Hammer and U. van Kolck, Nucl. Phys. A **676**, 357 (2000).
 - [8] H.-W. Hammer and T. Mehen, Phys. Lett. B **516**, 353 (2001).
 - [9] P. F. Bedaque and H. W. Griesshammer, Nucl. Phys. A **671**, 357 (2000).
 - [10] F. Gabbiani, P. F. Bedaque and H. W. Griesshammer, Nucl. Phys. A **675**, 601 (2000).
 - [11] J.-W. Chen, G. Rupak, M.J. Savage, Phys. Lett. B **464**, 1 (1999).
 - [12] G. Rupak, Nucl. Phys. A **678**, 405 (2000).
 - [13] S. Ando, R. H. Cyburt, S. W. Hong, C. H. Hyun, Phys. Rev. C **74**, 025809 (2006).
 - [14] P. F. Bedaque, G. Rupak, H. W. Griesshammer and H.-W. Hammer, Nucl. Phys. A **714**, 589 (2003).
 - [15] H. Sadeghi and S. Bayegan, Nucl. Phys. A **753**, 291 (2005).
 - [16] ENDF/B online database at the NNDC Online Data Service, <http://www.nndc.bnl.gov>.
 - [17] W. Dilg, L. Koester and W. Nistler, Phys. Lett. B **36**, 208 (1971).
 - [18] H. W. Griesshammer, Nucl. Phys. A **744**, 192 (2004).
 - [19] L.E. Marcucci, M. Viviani, R. Schiavilla, A. Kievsky and S. Rosati, Phys. Rev. C **72** (2005), 014001
 - [20] H.W. Griesshammer, Nucl. Phys. A **760** (2005), 110